

Rank preserving Structural Nested Distribution Model (RPSNDM) for Continuous Y :

$$Y_{\bar{a}=0} = Y_{\bar{a}} - \psi^* \sum_{m=0}^M a_m = Y_{\bar{a}} - \psi^* cum(\bar{a}).$$

$$Y_{\bar{a}} = Y_{\bar{a}=0} + \psi^* cum(\bar{a})$$

ψ^* an unknown parameter.

$$\psi^* = 0 \Leftrightarrow Y_{\bar{a}} = Y_{\bar{a}=0} = Y \text{ for all subjects}$$

Rank Preserving (RP): $Y_{\bar{a}=0,i} > Y_{\bar{a}=0,j}$ then $Y_{\bar{a},i} > Y_{\bar{a},j}$ for any \bar{a} .
More generally

$$Y_{\bar{a}=0} = h(Y_{\bar{a}}) - \psi^* \sum_{m=0}^M a_m = Y_{\bar{a}} - \psi^* cum(\bar{a}).$$

$$Y_{\bar{a}} = Y_{\bar{a}=0} + \psi^* cum(\bar{a})$$

Remark: RP biologically implausible and pedagogically useful.(and mathematically unnecessary if no unmeasured confounders but not for instrumental variables)

Knowledge of ψ^* gives *CAN* estimates of $\text{dist } Y_{\bar{a}=0}$ and $Y_{\bar{a}}$

$$Y_{\bar{a}=0} = Y(\psi^*) \equiv Y_A - \psi^* \text{cum}(\bar{A}) = Y - \psi^* \text{cum}(\bar{A})$$

$\text{pr}[Y_{\bar{a}=0} < t] = \text{pr}[Y(\psi^*) < t]$ estimated by the fraction of subjects with $Y(\psi^*)$ less than t . That is

$$\hat{\text{pr}}[Y_{\bar{a}=0} < t] = n^{-1} \sum_i I[Y(\psi^*) < t]$$

, where $I[B] = 1$ if B is true and $I[B] = 0$ if B is false.

Since $Y_{\bar{a}=0} = Y(\psi^*)$, it follows $Y_{\bar{a}=0}$

$$Y_{\bar{a}} = Y(\psi^*) + \psi^* \text{cum}(\bar{a})$$

$\text{pr}[Y_{\bar{a}} < t] = \text{pr}[Y(\psi^*) + \psi^* \text{cum}(\bar{a}) < t]$ estimated by the fraction of subjects with $Y(\psi^*) + \psi^* \text{cum}(\bar{a})$ less than t .

$$\hat{\text{pr}}[Y_{\bar{a}} < t] = n^{-1} \sum_i I[Y(\psi^*) + \psi^* \text{cum}(\bar{a}) < t]$$

Only remains to estimate ψ^* .

Remark: Above true without assuming no unmeasured confounders, i.e sequential randomization. Assumption of no unmeasured confounders critical for estimating ψ^*

G-Estimation: Under sequential randomization, (no unmeasured confounders)

$$Y_{\bar{a}=0} \Pi A_m | \bar{L}_m, \bar{A}_{m-1}$$

$\log it pr [A_m = 1 | \bar{L}_m, \bar{A}_{m-1}, Y_{\bar{a}=0}] = \alpha W_m + \theta Y_{\bar{a}=0}$ has $\theta = 0$

and thus

$\log it pr [A_m = 1 | \bar{L}_m, \bar{A}_{m-1}, Y(\psi^*)] = \alpha W_m + \theta Y(\psi^*)$ has $\theta = 0$.

We can calculate for each ψ

$$Y(\psi) = Y_{\bar{A}} - \psi cum(\bar{A}) = Y - \psi cum(\bar{A})$$

and fit $\log it pr [A_m = 1 | \bar{L}_m, \bar{A}_{m-1}, Y(\psi)] = \alpha W_m + \theta Y(\psi)$

Looks backward, but recall that, unlike Y , $Y(\psi^*) = Y_{\bar{a}=0}$ is baseline but (unmeasured).

Given our RPSNDM and no unmeasured confounders we can estimate each persons $Y(\psi^*) = Y_{\bar{a}=0}$!!!

$\hat{\psi}$ is the value of ψ such that the score, Wald or LR test of $\theta = 0$ is 0 (p-value is 1) when using $Y(\hat{\psi})$ in the logistic regression.

ψ in 95% CI \Leftrightarrow score, Wald or LR .05 level test of $\theta = 0$ accepts when using $Y(\psi)$ in the logistic regression

In practice search over a grid of values $\psi = -1, -.9, -.8, \dots, -.1, 0, .1, \dots, .9, 1$

Interactions with Time-Dependent Covariates:

$$Y_{\bar{a}=0} = Y_{\bar{a}} - \psi_1^* \sum_{m=0}^M a_m - \psi_2^* \sum_{m=0}^M a_m L_{\bar{a},m}^*$$

ψ_1^* and $\psi_1^* + \psi_2^*$ different signs = qualitative interaction.

$$Y_{\bar{a}=0} = Y(\psi^*) \equiv Y - \psi_1^* cum(\bar{A}) - \psi_2^* \sum_{m=0}^M A_m L_m^*$$

$$\log itpr[A_m = 1 \mid \bar{L}_m, \bar{A}_{m-1}, Y(\psi)] \quad ((1))$$

$$= \alpha W_m + \theta_1 Y(\psi) + \theta_2 Y(\psi) L_m^* \quad ((2))$$

$\psi = (\psi_1, \psi_2)$ in 95% CI \Leftrightarrow .05 level score, Wald, LR test of $\theta_1 = \theta_2 = 0$ accepts.

More generally

$$\begin{aligned} Y_{\bar{a}=0} &= h(Y_{\bar{a}}, \bar{L}_{\bar{a},m}, \psi^*) \\ &= Y_{\bar{a}} - \psi_1^* \sum_{m=0}^M a_m - \psi_2^* \sum_{m=0}^M a_m L_{\bar{a},m}^* \\ Y_{\bar{a}=0} &= Y(\psi^*) \equiv h(Y, \bar{L}, \psi^*) = Y - \psi_1^* cum(\bar{A}) - \psi_2^* \sum_{m=0}^M A_m L_m^* \end{aligned}$$

Instrumental Variables:

$$A_m = (A_{pm}, A_{dm}),$$

A_{pm} = *prescribed* treatment

A_{dm} = *received* treatment

$$Y_{\bar{a}} \perp\!\!\!\perp A_{pm} \mid \bar{L}_m, \bar{A}_{m-1}$$

$$\log \text{itpr} [A_{pm} = 1 \mid \bar{L}_m, \bar{A}_{m-1}, Y(\psi)] = \alpha W_m + \theta Y(\psi)$$

Sufficient Condition: On causal DAG no U that is a cause of Y has an arrow directly into A_{pm} but now may have into A_{dm}

Def: If exclusion restriction that

$$Y_{\bar{a}=0} = h(Y_{\bar{a}}, \psi^*) = Y_{\bar{a}} - \psi^* \sum_{m=0}^M a_{d,m} = Y_{\bar{a}} - \psi^* \text{cum}(\bar{a}_d)$$

does not depend on \bar{a}_p holds, we say A_{pm} is an instrumental variable for the effect of A_{dm} on Y

On causal Dag no directed paths from A_{pm} to Y except through $A_{dk}, k \geq m$

Simplest Example: Randomized Trial with Non Compliance

$$Y_{\bar{a}=0} = Y_{\bar{a}} - \psi^* a_d, Y(\psi) = Y - \psi A_d$$

Score test of $\theta = 0$ in model

$$\log \text{itpr} [A_p = 1 | Y(\psi)] = \alpha + \theta Y(\psi)$$

with $e^\alpha / (1 + e^\alpha) = 1/2$ is

$$\sum_i Y(\psi) (A_p - 1/2) = \sum_i (Y - \psi A_d) (A_p - 1/2)$$

Thus

$$\hat{\psi} = \frac{\sum_i Y_i (A_{p,i} - 1/2)}{\sum_i A_{d,i} (A_{p,i} - 1/2)} = \frac{\hat{E}[Y | A_p = 1] - \hat{E}[Y | A_p = 0]}{\hat{E}[A_d | A_p = 1] - \hat{E}[A_d | A_p = 0]}$$

If

$$\widehat{\psi} = \frac{\sum_i Y_i (A_{p,i} - 1/2)}{\sum_i A_{d,i} (A_{p,i} - 1/2)} = \frac{\widehat{E}[Y|A_p = 1] - \widehat{E}[Y|A_p = 0]}{\widehat{E}[A_d|A_p = 1] - \widehat{E}[A_d|A_p = 0]}$$

Note

$$\begin{aligned} & \frac{E[Y|A_p = 1] - E[Y|A_p = 0]}{E[A_d|A_p = 1] - E[A_d|A_p = 0]} \\ = & \frac{E[Y_0 + \psi A_d|A_p = 1] - E[Y_0 + \psi A_d|A_p = 0]}{E[A_d|A_p = 1] - E[A_d|A_p = 0]} \\ = & \frac{E[Y_0|A_p = 1] - E[Y_0|A_p = 0]}{E[A_d|A_p = 1] - E[A_d|A_p = 0]} \\ & + \frac{\psi E[A_d|A_p = 1] - \psi E[A_d|A_p = 0]}{E[A_d|A_p = 1] - E[A_d|A_p = 0]} \\ = & \psi \end{aligned}$$

$E[A_d|A_p = 1] = E[A_d|A_p = 0]$, then under exclusion restriction this will estimate $\frac{0}{0}$, i.e. undefined rather than ψ^* . So need A_p and A_d correlated

Difficulties with Instruments:

Finding an instrument in observational studies.

Famous example Vietnam service and suicide from draft lottery.

True Observational Instrument : Y is survive heart attack.

Does SG influence survival after MI

Distance (A_p) from Hospital where they could do Swan Ganz ?.

Idea is A_p has no direct effect so unrelated to survival under the null Y_0 , but ambulance more likely to take you to the SG hospital if closer and thus you get more SG.

Ambulance driver thinks guy looks sick and says "Lets go the little extra to get him to the SG hospital. This way SG hospital gets worse patients. Is this bias solved by our instrument.?"

But profound problems:

1. Problems: Is only difference between hospitals SG.

Suppose risk better in SG but every subject who is cathed dies. Must have causal pathway from SG to survival not through SG if no unmeasured confounding for A_p . Indeed this allows a test (not consistent) of the exclusion restriction

Related to ecologic fallacy. More suicides where more protestants but actually these are among the catholics.

Neighborhoods differ in risk and distance to hospital so maybe A_p is related to survival under the null Y_0

Weak instrument : effect of unmeasured confounders or of failure of exclusion restriction blown way up

Model Dependence: What if constant treatment effect (rank preservation)
wrong as must be if Y is dichotomous and an effect is present. Does

$$\hat{\psi} = \frac{\sum_i Y_i (A_{p,i} - 1/2)}{\sum_i A_{d,i} (A_{p,i} - 1/2)} = \frac{\hat{E}[Y|A_p = 1] - \hat{E}[Y|A_p = 0]}{\hat{E}[A_d|A_p = 1] - \hat{E}[A_d|A_p = 0]}$$

estimate something meaningful if A_p is an instrument (no unmeasured confounders and exclusion restriction)

If A_p and A_d dichotomous and one more condition the answer is yes .

It estimates the average treatment effect among the compliers.

$A_{d,A_p=1} = A_{d,1}$ and $A_{d,A_p=0} = A_{d,0}$ counterfactuals for received treatment given assigned treatment .

- $(A_{d,0}, A_{d,1}) = (0, 0)$ never taker
- $(A_{d,0}, A_{d,1}) = (1, 1)$ always taker
- $(A_{d,0}, A_{d,1}) = (0, 1)$ complier
- $(A_{d,0}, A_{d,1}) = (1, 0)$ defier

Pretreatment definitons:

Theorem (Imbens and Angrist , 1994): If exclusion and no unmeasured confounders hold for A_p and no defiers (nonidentifiable except if no tskers in control- $A_p = 0$) then

$$\psi^* = \frac{E[Y|A_p = 1] - E[Y|A_p = 0]}{E[A_d|A_p = 1] - E[A_d|A_p = 0]}$$

is $E[Y_{(1,1)} - Y_{1,0} | (A_{d,0}, A_{d,1}) = (0, 1)]$.

Idea: Only can estimate the effect of treatment on those the instrument actually changed their treatment so compliers and defiers only relevant sets. If both gt mixed up. If only compliers OK.

Remark: No model (RP) dependence. Still sometimes can empirically rule out exclusion restriction or no unmeasured confounders as before.

Bounds for $ATE = E [Y_{(1,1)} - Y_{1,0}]$ without RP under A_p an instrument

See Robins Greenland (1996) discussion of Angrist Imbens Rubin JASA

Note now not identified even if we assume no defiers.

For example suppose $\psi^* = \frac{E[Y|A_p=1] - E[Y|A_p=0]}{E[A_d|A_p=1] - E[A_d|A_p=0]} = 0$, how big small can $E [Y_{(1,1)} - Y_{1,0}]$ be even without defiers.

Idea is those people treatment good for (bad for) were never takers or always taskers.

What does this imply philosophically about ITT analyses?

Rank Preserving Structural Nested failure Time Models

If $\bar{a} = a$ does not change in time natural to use accelerated failure time model.

$$\begin{aligned} T_{\bar{a}} &= T_{\bar{a}=0} e^{-\alpha \psi^*} \\ T_{\bar{a}=0} &= T_{\bar{a}} e^{\alpha \psi^*} \\ \log T_{\bar{a}} &= \ln T_{\bar{a}=0} - \alpha \psi^* \\ \log T &= -\psi^* A + \varepsilon, \varepsilon = \log T_{\bar{a}=0} \\ \log T &= \alpha - \psi^* A + \varepsilon, \alpha = E[\varepsilon]. \end{aligned}$$

$\psi^* = 0$ implies $T_{\bar{a}} = T_{\bar{a}=0} = T$.

$\psi^* > 0$ implies exposure harmful ($T_{\bar{a}} < T_{\bar{a}=0}$).

$\psi^* < 0$ implies exposure beneficial ($T_{\bar{a}} > T_{\bar{a}=0}$).

$\text{median}(T_{\bar{a}}) / \text{median}(T_{\bar{a}=0}) = e^{-\alpha \psi^*}$

Proof: :

$$\begin{aligned} .5 &= \text{pr} \{T_{\bar{a}} < \text{median}(T_{\bar{a}})\} \\ &= \text{pr} \left\{ T_{\bar{a}} e^{\alpha \psi^*} < e^{\alpha \psi^*} \text{median}(T_{\bar{a}}) \right\} = \text{pr} \left\{ T_{\bar{a}=0} < e^{\alpha \psi^*} \text{median}(T_{\bar{a}}) \right\} \end{aligned}$$

so $\text{median}(T_{\bar{a}=0}) = e^{\alpha \psi^*} \text{median}(T_{\bar{a}})$. Hence $\text{median}(T_{\bar{a}}) / \text{median}(T_{\bar{a}=0}) = 1/e^{\alpha \psi^*}$

Generalization to time-dependent exposures $\bar{a} = (a_0, a_1, \dots, a_k, \dots)$ considered constant at a_m between m and $m+1$ so $a(t)$ is defined for all $t \geq 0$.

RPSFTM: $T_{\bar{a}=0}$ is the area under the curve $g(t) = \exp(\psi^* a(t))$ from 0 to $T_{\bar{a}}$

$$T_{\bar{a}=0} = \int_0^{T_{\bar{a}}} \exp(\psi^* a(t)) dt = \sum_{m=0}^{\text{int}[T_{\bar{a}}]-1} e^{\psi^* a_m} + \{T_{\bar{a}} - \text{int}[T_{\bar{a}}]\} e^{\psi^* a_{\text{int}[T_{\bar{a}}]}}$$

where $\text{int}[x]$ is the largest integer less than x . $\text{int}[3.2] = 3$.

Time constant $\bar{a} : a_m = a$ for all m .

$$\begin{aligned} T_{\bar{a}=0} &= \exp(\psi^* a) \int_0^{T_{\bar{a}}} dt = T_{\bar{a}} \exp(\psi^* a) \\ T_{\bar{a}} &= \exp(-\psi^* a) T_{\bar{a}=0} \end{aligned}$$

Knowledge of ψ^* gives estimates of $\text{dist } T_{\bar{a}=0}$ and $T_{\bar{a}}$

$$T_{\bar{a}=0} = T(\psi^*) \equiv \int_0^T \exp(\psi^* A(t)) dt = \sum_{m=0}^{int[T]-1} e^{\psi^* A_m} + \{T - int[T]\} e^{\psi^* A_{int[T]}}$$

$\text{pr}[T_{\bar{a}=0} < t] = \text{pr}[T(\psi^*) < t]$ estimated by the fraction of subjects with $T(\psi^*)$ less than t . That is

$$\hat{pr}[T_{\bar{a}=0} < t] = n^{-1} \sum_i I[T(\psi^*) < t]$$

, where $I[B] = 1$ if B is true and $I[B] = 0$ if B is false.

Since $T_{\bar{a}=0} = T(\psi^*)$, it follows for time unvarying $\bar{a} : a_m = a$ for all m .

$$T_{\bar{a}} = \exp(-\psi^* a) T(\psi^*)$$

$\text{pr}[T_{\bar{a}} < t] = \text{pr}[\exp(-\psi^* a) T(\psi^*) < t]$ estimated by the fraction of subjects with $\exp(-\psi^* a) T(\psi^*)$ less than t .

$$\hat{pr}[T_{\bar{a}} < t] = n^{-1} \sum_i I[\exp(-\psi^* a) T(\psi^*) < t]$$

Only remains to estimate ψ^* .

G-Estimation: Under sequential randomization (no unmeasured confounders)

$$T_{\bar{a}=0} \equiv A_m | \bar{L}_m, \bar{A}_{m-1}$$

$\log it\ pr [A_m = 1 | \bar{L}_m, \bar{A}_{m-1}, T_{\bar{a}=0}] = \alpha W_m + \theta T_{\bar{a}=0}$ has $\theta = 0$

and thus

$\log it\ pr [A_m = 1 | \bar{L}_m, \bar{A}_{m-1}, T(\psi^*)] = \alpha W_m + \theta T(\psi^*)$ has $\theta = 0$.

We can calculate for each ψ

$$T(\psi) \equiv \int_0^T \exp(\psi A(t)) dt = \sum_{m=0}^{int[T]-1} e^{\psi A_m} + \{T - int[T]\} e^{\psi A_{int[T]}}$$

and fit $\log it\ pr [A_m = 1 | \bar{L}_m, \bar{A}_{m-1}, T(\psi)] = \alpha W_m + \theta T(\psi)$

$\hat{\psi}$ is the value of ψ such that the score, Wald or LR test of $\theta = 0$ is 0 (p-value is 1) when using $T(\hat{\psi})$

in the logistic regression.

ψ in 95% CI \Leftrightarrow score, Wald or LR .05 level test of $\theta = 0$ accepts when using $T(\psi)$ in the logistic regression

In practice search over a grid of values $\psi = -1, -.9, -.8, \dots, .1, 0, .1, \dots, .9, 1$.

Interactions with Time-Dependent Covariates:

$$T_{\bar{a}=0} = T(\psi^*) = \int_0^{T_{\bar{a}}} \exp(\psi_1^* a(t) + \psi_2^* a(t) L^*(t)) dt$$

ψ_1^* and $\psi_1^* + \psi_2^*$ different signs = qualitative interaction.

$$T_{\bar{a}=0} = T(\psi^*) = \int_0^T \exp(\psi_1^* A(t) + \psi_2^* A(t) L^*(t)) dt$$

$$\log \text{itpr}[A_m = 1 \mid \bar{L}_m, \bar{A}_{m-1}, T(\psi)] \quad ((1))$$

$$= \alpha W_m + \theta_1 T(\psi) + \theta_2 T(\psi) L_m^* \quad ((2))$$

$\psi = (\psi_1, \psi_2)$ in 95% CI \Leftrightarrow .05 level score, Wald, LR test of $\theta_1 = \theta_2 = 0$ accepts.

$\hat{\psi} = \psi = (\psi_1, \psi_2) \Leftrightarrow$ score, Wald, LR test of $\theta_1 = \theta_2 = 0$ is 0

Relation of SNFTM without interactions with time varying covariates to Cox Proportional MSMs

If $T_{\bar{a}=0}$ has an exponential distribution (i.e the hazard of $T_{\bar{a}=0}$ is a constant λ_0 [i.e., $\lambda_{T_{\bar{a}=0}}(t) = \lambda_0$] that does not change with time) and no interactions, then

$$\lambda_{T_{\bar{a}}}(t) = \lambda_{T_{\bar{a}=0}}(t) \exp(\psi^* a(t))$$

so our SNFTM is also a MSM with the same parameter.

Results SNFTM analysis of MAC: $\exp(\hat{\psi}) = .6$ so, assuming $T_{\bar{a}=0}$ has an exponential distribution, causal hazard ratio estimate of .6

Recall MSM causal hazard ratio estimate was .6

Test of whether T_0 has an exponential distribution by testing whether the $T_{\bar{a}=0,i}(\hat{\psi}); i = 1, \dots, n$ is consistent with a sample from an exponential distribution .

Advantages of MSMs:

1. Easier to understand
2. No g-estimation of logistic SNMs so cannot be used for dichotomous non rare outcomes. For discrete data, g-estimation of SN mean models with linear and log linear links only.
3. Can estimate nondynamic regimes \bar{a} (regimes where treatment at m does not depend on past covariate history \bar{L}_m , even if there is interaction with time-dependent covariates)

Advantages of SNMs:

1. Estimate Interaction with time-dependent covariates
2. Estimation of dynamic regimes
3. Does not need the positivity assumption that $pr(A_m = 1 | \bar{L}_m, \bar{A}_{m-1}) > 0$ for all \bar{L}_m, \bar{A}_{m-1} histories that occur.
Example Occupational health cohort study .
4. Can use instrumental variables

Censoring by Administrative End Of Follow Up in SNFTM

C_i – the potential administrative censoring time is time from start of follow-up for a subject to administrative end of follow up defined for purposes of analysis.

C_i – is known at baseline for all subjects even those who fail, in contrast to censoring by competing risks.

Assume A_m is either 0 or 1

Assume administrative censoring is the only form of censoring (i.e., there is no censoring by lost to follow-up or competing risks.)

Let $\Delta = I(C > T)$ be the indicator of observing failure on a subject

If $\Delta = 0$ the subject will be administratively censored.

$T(\psi)$ is not observed for administratively censored subjects.

Therefore in g-estimation we replace $T(\psi)$ by either of the always observed variables $X(\psi)$ or $\Delta(\psi)$

where

$$\begin{aligned}\Delta(\psi) &= I(T(\psi) < C(\psi)), X(\psi) = \min(T(\psi), C(\psi)) \\ C(\psi) &= C \text{ if } \psi \geq 0 \text{ and } C(\psi) = Ce^\psi \text{ if } \psi < 0\end{aligned}$$

Note we think of $\Delta(\psi)$ as an artificial censoring variable since when $\psi \neq 0$, $\Delta(\psi)$ may be zero even for an observed failure ($\Delta = 1$). For example if $\psi > 0$, this will occur whenever

$$T < C = C(\psi) < T(\psi) = \int_0^T \exp(\psi A(t)) dt$$

To prove $X(\psi)$ and $\Delta(\psi)$ are always observed note

(1) C and thus $C(\psi)$ is always observed and

(2) whenever $C < T$ so T and $T(\psi)$ are unobserved,

$C(\psi) < T(\psi)$ since

(a) $C(\psi) = C \leq \int_0^C \exp(\psi A(t)) dt T(\psi) \leq \int_0^T \exp(\psi A(t)) dt = T(\psi)$ when $\psi \geq 0$ and

(b) $C(\psi) = Ce^\psi \leq \int_0^C \exp(\psi A(t)) dt \leq \int_0^T \exp(\psi A(t)) dt = T(\psi)$ when $\psi < 0$

Hence $X(\psi) = \min(T(\psi), C(\psi))$ and $\Delta(\psi) = I(T(\psi) < C(\psi))$ are always observed.

Further since C is known at baseline, by definition, C is a component of the baseline measured covariates $L(0)$.

Thus $X(\psi)$ and $\Delta(\psi)$ are functions of $T(\psi)$ and L_0 . Thus

$$T_{\bar{a}=0} \parallel A_m | \bar{L}_m, \bar{A}_{m-1}, C$$

implies

$$T(\psi^*) \parallel A_m | \bar{L}_m, \bar{A}_{m-1}, C$$

which implies

$$(X(\psi^*), C(\psi^*)) \Pi A_m | \bar{L}_m, \bar{A}_{m-1}, C$$

which justifies G-estimation to estimate ψ^* based on either of the observed variables $X(\psi)$ or $\Delta(\psi)$ replacing the not always observed $T(\psi)$.

We adjust for any additional censoring due to loss to follow-up or competing risks, by weighting the day m specific contribution of an uncensored surviving subject contributing to g-estimation of A_m with

the inverse probability of remaining uncensored by loss to follow-up (and /or competing risks) from time m till the time $\min(C_i, T_i)$ [the time at which $X(\psi)$ and $\Delta(\psi)$ become computable and so can be entered in the logistic model for A_m .]